Two Way Slab

Problem Statement:

Use the ACI 318 Direct Design Method to design an interior bay of a flat plate slab system of multi bay building. The Dimensions of an interior bay are shown in Figure 1.

Figure 1. Interior Bay Dimensions

Two-way Slab
Design Data

Column Size - 24” x 24”

Materials
• Concrete: normal weight (150 pcf), f’c = 4,000 psi
• Welded Wire Reinforcement (WWR), fy = 80,000 psi

Loads
• Superimposed dead loads = 30 psf
• Live load = 50 psf

Slab Thickness
Longest clear span ln = 24 – (24/12) = 22.0 ft
Minimum thickness h per ACI Table 9.5(c) = ln/30 = 8.8 in. Use h 9.0 in.

q_u = 1.2(9/12*150 + 30) + 1.6(50) = 251.0 psf

Design for Flexure
Check if the Direct Design Method of ACI Sect. 13.6 can be utilized to compute the bending moments due to the gravity loads:
• 3 continuous spans in one direction, more than 3 in the other O.K.
• Rectangular panels with long-to-short span ratio = 24/20 = 1.2 < 2 O.K.
• Successive span lengths in each direction are equal O.K.
• No offset columns O.K.
• L/D = 50/(112.5 + 30) = 0.35 < 2 O.K.

Since all requirements are satisfied, the Direct Design Method can be used.

Short Direction (20 ft)
l_2 = 24 ft and l_n = 20’-2’ = 18.0 ft
Factored moment per span.

Mo = \frac{q_u l_2 l_n^2 h}{8} = \frac{0.251\cdot24\cdot18}{8} = 244 \text{ ft} - \text{kips}

Division of the total panel moment Mo into negative and positive moments, and then column and middle strip moments, involves the direct application of the moment coefficients in the following Table 1.
This example is to be used for educational purposes only.

Table 1 Moment Distribution.

<table>
<thead>
<tr>
<th>Slab Moment</th>
<th>Int. Neg.</th>
<th>Int. pos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Moment</td>
<td>170.80</td>
<td>85.40</td>
</tr>
<tr>
<td>Column Strip</td>
<td>128.10</td>
<td>51.24</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>42.70</td>
<td>34.16</td>
</tr>
</tbody>
</table>

Reinforcement,
Assume D16 wires will be used.
d = 9 - 0.75 -0.45/2 = 8.03 in.
Column strip width b = (20 x 12)/2 = 120 in.
Middle strip width b = (24 x 12) – 120 = 168 in.
As = Mu /4d*60/80 where Mu is in ft-kips and d is in inches
Min. As = 0.0018bh*60/80 (grade 80,000 psi steel)
Max. s = 2h = 18 in. or 18 in. (Sect. 13.3.2)
The required reinforcements are shown in table 2.

<table>
<thead>
<tr>
<th>Span Location</th>
<th>Mu kips-ft</th>
<th>b (in)</th>
<th>d (in)</th>
<th>As (in2)</th>
<th>Min As (in2)</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column Strip</td>
<td>Int. Neg.</td>
<td>128.10</td>
<td>120.00</td>
<td>8.03</td>
<td>2.99</td>
<td>1.46</td>
</tr>
<tr>
<td>Int. Pos.</td>
<td>51.24</td>
<td>120.00</td>
<td>8.03</td>
<td>1.20</td>
<td>1.46</td>
<td>D8 at 6 in.</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>Int. Neg.</td>
<td>42.70</td>
<td>168.00</td>
<td>8.03</td>
<td>1.00</td>
<td>2.04</td>
</tr>
<tr>
<td>Int. Pos.</td>
<td>34.16</td>
<td>168.00</td>
<td>8.03</td>
<td>0.80</td>
<td>2.04</td>
<td>D8 at 6 in.</td>
</tr>
</tbody>
</table>

Table 2. Required Bay Reinforcement.

**Long Direction (24 ft)**

l₂ = 20 ft and lₙ = 24-2= 22.0 ft

Factored moment per span.

\[ Mo = \frac{ql₂lₙ²}{8} = \frac{0.251 \times 20 \times 22²}{8} = 304.0 \text{ ft} - \text{kips} \]

Division of the total panel moment Mo into negative and positive moments, and then column and middle strip moments, involves the direct application of the moment coefficients in the following Table 3.

<table>
<thead>
<tr>
<th>Slab Moment</th>
<th>Int. Neg</th>
<th>Int. Pos.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total moment</td>
<td>212.10</td>
<td>106.05</td>
</tr>
<tr>
<td>Column Strip</td>
<td>159.08</td>
<td>63.63</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>53.03</td>
<td>42.42</td>
</tr>
</tbody>
</table>

Table 3 Moment Distribution.
Reinforcement,
Assume D20 wires will be used.

d = 9 - 0.75 - 0.45 - 0.5/2 = 7.55 in.

*Column strip width b = (20 x 12)/2 = 120 in.*

*Middle strip width b = (20 x 12) – 120 = 120 in.*

As = $\frac{Mu}{4d*60/80}$ where $Mu$ is in ft-kips and $d$ is in inches

*Min. As = 0.0018bh*60/80 (grade 80 ksi )

*Max. s = 2h = 18 in. or 18 in. (Sect. 13.3.2)*

The required reinforcements are shown in table 4.

<table>
<thead>
<tr>
<th>Span Location</th>
<th>Mu kips-ft</th>
<th>b (in)</th>
<th>d (in)</th>
<th>As (in²)</th>
<th>Min As (in²)</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column Strip</td>
<td>Int. Neg.</td>
<td>159.08</td>
<td>120.00</td>
<td>7.55</td>
<td>3.95</td>
<td>1.46</td>
</tr>
<tr>
<td></td>
<td>Int. Pos.</td>
<td>63.63</td>
<td>120.00</td>
<td>7.55</td>
<td>1.58</td>
<td>1.46</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>Int. Neg.</td>
<td>53.03</td>
<td>120.00</td>
<td>7.55</td>
<td>1.32</td>
<td>1.46</td>
</tr>
<tr>
<td></td>
<td>Int. Pos.</td>
<td>42.42</td>
<td>120.00</td>
<td>7.55</td>
<td>1.05</td>
<td>1.46</td>
</tr>
</tbody>
</table>

Table 4. Required Bay Reinforcement.

Reinforcement Details:

The following figures 2, 3, 4 and 5 show the reinforcement details for the column and middle strips for both directions. The lengths were determined from Figure 13.3.8 of ACI 318-05.
Bottom Reinforcement

Figure 2. Top and Bottom Reinforcement
Figure 3. Sheets Details

Column Strip

This example is to be used for educational purposes only.
Middle Strip

Figure 4. Cross Section (Short Direction)
Figure 5. Cross Section (Long Direction)
Reinforcement Quantities:

<table>
<thead>
<tr>
<th>Sheet</th>
<th>Number and Size</th>
<th>Length (ft)</th>
<th>Weight per Foot (lb)</th>
<th>Weight (lb)</th>
<th>Total Weight of Sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>WWR1</td>
<td>11 D16</td>
<td>11</td>
<td>0.544</td>
<td>65.8</td>
<td>189.6</td>
</tr>
<tr>
<td></td>
<td>14 D20</td>
<td>13</td>
<td>0.68</td>
<td>123.8</td>
<td></td>
</tr>
<tr>
<td>WWR2</td>
<td>4 D8</td>
<td>9</td>
<td>0.272</td>
<td>9.8</td>
<td>66.9</td>
</tr>
<tr>
<td></td>
<td>18 D8</td>
<td>11.67</td>
<td>0.272</td>
<td>57.1</td>
<td></td>
</tr>
<tr>
<td>WWR3</td>
<td>22 D8</td>
<td>10</td>
<td>0.272</td>
<td>59.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 D8</td>
<td>11</td>
<td>0.272</td>
<td>12.0</td>
<td>71.8</td>
</tr>
<tr>
<td>WWR4</td>
<td>52 D8</td>
<td>10.67</td>
<td>0.272</td>
<td>150.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>21 D8</td>
<td>26</td>
<td>0.272</td>
<td>148.5</td>
<td>299.4</td>
</tr>
</tbody>
</table>

Each 20’ by 24’ bay has one WWR1, one WWR2, one WWR3 and two WWR4. The total steel weight for each bay = 189.6+66.9+71.8+2*299.4 = 927.1 lbs.
Steel weight per square foot = 927.1 / (24*20) = 1.93 lb/ft^2

Design Using Grade 60 Rebar.

Figures 6 and 7 show the reinforcement detail of the interior bay using Grade 60 Rebar.
The total steel weight for each bay using Grade 60 Rebar = 1,204 lbs.
Steel weight per square foot = 1,204/(24*20) = 2.51 lb/ft^2

Conclusions:
The ratio between the total weight per square foot using WWR to the total weight per square foot using Grade 60 Rebar = (1.93/2.51) *100 = 77%.
Placed of WWR Should be significantly faster than placement of the reinforcing bars, resulting in additional overall saving.
Figure 6. Cross Section (Short Direction), Grade 60 Rebar
Figure 7. Cross Section (Long Direction), Grade 60 Rebar