SHEAR REINFORCEMENT OF A BRIDGE GIRDER

System Description: 130.0 ft. simple span, four NU1350 girder lines, spaced at 12 ft. o.c. Figure 1 shows a cross section of this system. The deck is 8 in. thick, and a future wearing surface of 2 in. is considered. NU1350

![Cross Section](image)

The girder concrete strength is 10.0 ksi and its web width is 5.91 in. Grade 80 ksi deformed WWR for shear reinforcement. The girder is reinforced by 60-0.6 in diameter low relaxation strands; all strands are straight and fully tensioned.

Required:

It is required to design for shear using AASHTO specification modified compression field theory.

Moment and Shear

Table 1 shows $M_u$ and $V_u$ through the length of the girder and at the critical sections.

<table>
<thead>
<tr>
<th>Section</th>
<th>0</th>
<th>Critical Section</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from the End</td>
<td>0</td>
<td>5</td>
<td>13</td>
<td>26</td>
<td>39</td>
<td>52</td>
<td>65</td>
</tr>
<tr>
<td>$M_u$</td>
<td>0</td>
<td>1617</td>
<td>3961.8</td>
<td>7027</td>
<td>9064</td>
<td>10120.8</td>
<td>10245.6</td>
</tr>
<tr>
<td>$V_u$</td>
<td>414.7</td>
<td>386</td>
<td>349.3</td>
<td>284.2</td>
<td>220.8</td>
<td>158.9</td>
<td>98.6</td>
</tr>
</tbody>
</table>

Table 1. Shear and Moment values
Shear Reinforcement

Shear design using the **Sectional Design Model** is an iterative process that begins by assuming a value for $\theta$.

Assume an initial value for the inclination of the compression field, $\theta$, of 25°.

Critical section for shear is greater of: (LRFD 5.8.3.2)

- $0.5 \, d_v \, \cot \theta$
- $d_v$

Compute $d_v$:

$d_v = \text{Effective shear depth} = \text{Distance between resultants of tensile and compressive forces}$

$a = 6.8 \text{ (from flexural analysis)}$

$d_v = d_e - \frac{a}{2} = \left( h_g + h_f - \text{c.g. @ critical section} \right) - \frac{a}{2} = \left( 53.1 + 7.5 -3.43 \right) - (6.8/2) = 53.77 \text{ in.}$

But $d_v$ need not be taken less than the greater of: (LRFD 5.8.2.7)

$0.9 \, d_e = (0.9) \, (60.6 -3.43) = 51.45 \text{ in.}$

$0.72 \, h = (0.72) \, (60.6) = 43.63 \text{ in.}$

Therefore, use $d_v = 53.77 \text{ in.}$

Therefore the critical section for shear is:

$0.50 \, \text{FT} + 53.77 \text{ in.} / 12 = 5.0 \text{ ft. from centerline of support.}$

At the critical section for shear, $V_u = 386.0 \text{ kips}$

**Component of Shear Resistance from Prestress, $V_p$**

Angle of centre of gravity of strand profile with respect to horizontal, $\alpha$:

$\alpha = 0$

$V_p = P_f \sin (\alpha) = 0$

**Governing Equations for Shear**

$V_u \leq V_r = \varphi V_n$ \hspace{1cm} (LRFD 5.8.2.1-2)

$\varphi = 0.90 \text{ for shear}$ \hspace{1cm} (LRFD 5.5.4.2.1)

$V_n = V_c + V_s + V_p$ \hspace{1cm} (LRFD 5.8.3.3-1)

Compute maximum shear capacity of section:

$V_{n_{max}} = 0.25 f'_c b_v d_v + V_p$ \hspace{1cm} (LRFD 5.8.3.3-2)
\[ V_{n,\text{max}} = (0.25) \times 10 \text{ KSI} \times 5.91 \text{ IN} \times 53.77 + 0 = 794 \text{ kips} \]

\[ \varphi V_{n,\text{max}} = (0.90) \times 794 = 715 \text{ kips} > V_u = 386 \text{ kips} \quad \text{O.K.} \]

**Concrete Contribution to Shear Resistance, \( V_c \)**

\[ \begin{align*}
V_c &= 0.0316 \beta \sqrt{f'_c} b_i d_v \\
V &= \frac{V_u - \varphi V_p}{\varphi b_i d_v} = \frac{386.0 \text{ KIP} - 0.9 \times 0.0 \text{ KIP}}{0.9 \times (5.91 \text{ IN}) \times (53.77 \text{ IN})} = 1.34 \text{ ksi} \\
\frac{V}{V_u} &= \frac{1.34 \text{ KSI}}{10.0 \text{ KSI}} = 0.134
\end{align*} \]

**Trial 1:** Assume \( \theta = 25^\circ \) (previously assumed and used to determine location of critical section for shear).

\[ \begin{align*}
M_u &= 1,617 \text{ K-FT} = 19404 \text{ K-in.} \quad \text{(see Summary of Dead and Live Load Effects)} \\
N_u &= 0 \quad \text{no applied axial loads} \\
f_{po} &= 0.7 \times 270 = 189.0 \text{ ksi} \\
A_{ps} &= \text{area of prestressing steel on flexural tension side of the member, i.e., the straight strands} \\
A_{ps} &= 13.02 \text{ in}^2 \\
\varepsilon_x &= \frac{M_u}{d_v} + 0.5 N_u + 0.5(V_u - V_p) \cot \theta - A_{ps} f_{po} \\
&= \frac{19404}{53.77} + 0.5(386 - 0) \cot(25^\circ) - (13.02)(189.0) \\
&= \frac{53.77}{2[(28.500)(13.02)]} = -0.00022
\end{align*} \]

Because \( \varepsilon_x \) is negative, use Eq. 5.8.3.4.2-3:

\[ \begin{align*}
\varepsilon_x &= \frac{M_u}{d_v} + 0.5 N_u + 0.5 V_u \cot \theta - A_{ps} f_{po} \\
&= \frac{2(E_c A_c + E_s A_s + E_p A_{ps})}{2(E_c A_c + E_s A_s + E_p A_{ps})} \varepsilon_x = -0.00003 = -0.03 \times 10^{-3}
\end{align*} \]

\[ A_c = \text{Area of concrete on flexural tension side} \\
= \text{Area of girder below h/2} = 62.1/2 = 31.05 \text{ in.} \\
= 450 \text{ in}^2 \]
From Table 5.8.3.4.2-1, with $\varepsilon_s = -0.0003 \times 10^{-3}$ and $\frac{V}{f'c} = 0.134$, find $\theta = 25.5^\circ$ and $\beta = 2.65$.

This is the same value as was assumed, so convergence has been achieved.

With these values, the concrete contribution, $V_c$, can now be computed.

$$V_c = 0.0316 \left(2.65\right)\sqrt{10.0 \text{ ksi}(5.91 \text{ in})(53.77 \text{ in})} = 84.15 \text{ kips}$$

**Required Shear Reinforcement, $V_s$ using WWR (80.0 ksi)**

Required $V_s = V_u / \varphi - V_c = 386 / 0.9 - 84.15 = 344.8$ kips

Assuming vertical stirrups,

$$V_s = \frac{A_v f_y d_v \cot \theta}{s}$$  \hspace{1cm} (LRFD 5.8.3.3-1)

Compute $A_v$ on an IN$^2$/FT basis ($s = 12$ in.):

$$A_v = \frac{12 V_s}{f_y d_v \cot \theta}$$

$$A_v = \frac{(12 \text{ in})(344.8 \text{ kips})}{(80 \text{ ksi})(53.77 \text{ in}) \cot(25.5^\circ)} = 0.45 \text{ in}^2/\text{ft}$$

Check minimum transverse reinforcement:

$$A_v = 0.0316 \sqrt{f'_c} \frac{b_v s}{f_y}$$  \hspace{1cm} (LRFD 5.8.2.5)

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$$A_v = 0.0316 \sqrt{f'_c} \frac{b_v s}{f_y}$$  \hspace{1cm} (LRFD 5.8.2.5)

$$A_v = 0.0316 \sqrt{10.00} \left(\frac{5.91}{80}\right) \left(\frac{12}{10.00}\right) = 0.088 \text{ in}^2/\text{ft} < 0.45 \text{ in}^2/\text{ft} \hspace{0.5cm} \text{O.K.}$$

Check maximum stirrup spacing:

$$V_u = 386 \text{ kips} > 0.1 f'_c b_v d_v = (0.1) (5.91) (10.00) (53.77) = 317.8 \text{ kips}$$

Therefore, maximum WWR spacing is 12 in.

For D12 WWR, maximum spacing is

$$s = 0.12 \times 2 \times 12 / 0.45 = 6.4 \text{ in.}$$

Use D12 WWR @ 6.0 in.

$$A_{v,prov'd} = 0.12(2)12/6 = 0.48 \text{ in}^2/\text{ft}$$

Table 2 shows the required shear reinforcement and spacing of the other sections.
Table 2. Required Shear Reinforcement and Spacing

<table>
<thead>
<tr>
<th>Distance from the End</th>
<th>0</th>
<th>5</th>
<th>13</th>
<th>26</th>
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<th>52</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>D12 @ in.</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

Figure 2 shows cross section and elevation of the girder with the WWR reinforcement. The details of WWR sheets are shown in Figure 3. Figure 4 shows Platte River East Bridge girder in Nebraska during fabrication.

Figure 2. Cross Section and Elevation of the Girder
WWR1

- 2 -D12
- D12@4"
- Bent here

WWR2

- 2 -D12
- D12@8"
- Bent here

This example is to be used for educational purposes only.
Figure 3 Sheet details

Figure 4. Platte River East Bridge girder in Nebraska during fabrication.
Reinforcement Quantities:

The total steel weight for the shear reinforcement = 542 lbs.

Design Using Grade 60 Rebar.

Figure 5 shows the reinforcement detail using Grade 60 Rebar. The total steel weight for the shear reinforcement using Grade 60 Rebar for the girder = 881 lbs.

Conclusions:

The ratio between the total steel weight using Grade 80 WWR to the total steel weight Grade 60 Rebar = (542/881)*100 = 62%. Placed of WWR Should be significantly faster than placement of the reinforcing bars, resulting in additional overall saving.

![Figure 5. Grade 60 Rebar Reinforcement](image)